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RELATIVISTIC MAGNETOHYDRODYNAMICS' EQUATIONS

(FRANCE)

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RELATIVISTIC MAGNETOHYDRODYNAMICS EQUATIONS *

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(Presented by M. Louis de Broglie)

Let us consider a medium of conductivity σ whose particles move with a velocity ν in an electromagnetic field $\vec{E}, \vec{H}$. The density of the conduction current at any point of the medium is given by

$$\vec{j} = \sigma\left(\vec{E} + \frac{1}{c}\left[\vec{v}, \vec{H}\right]\right). \qquad (1)$$

The Euler equations, which constitute the fundamental equations of nonrelativistic magnetohydrodynamics are written

$$\rho \frac{d\vec{v}}{dt} = -\operatorname{grad} p + \rho \vec{g} + \vec{F} + \frac{1}{c}\left[\vec{j}, \vec{H}\right], \qquad (2)$$

where $\vec{g}$ is the gravitational acceleration. In case of a viscous fluid,

$$\vec{F} = \rho \nu \Delta \vec{v}, \qquad (3)$$

where ν is the kinematic viscosity.

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* Les équations de la magnétohydrodynamique relativiste.-

Passing to the corresponding relativistic equations, we introduce after Mandelstamm-Tamm [1] the conductivity tensor $\sigma_{\alpha\beta\gamma}$ having the property that in the system of momentary rest, the components $\sigma_{114}, \sigma_{224}, \sigma_{334}$ are the only ones different from zero and equal to

$$\sigma_{114} = \sigma_{224} = \sigma_{334} = \sigma. \qquad (4)$$

Relative to a Lorentz transformation, the $\sigma_{\alpha\beta\gamma}$ are transformed according to the law

$$\sigma'_{\mu\nu\rho} = a_{\mu\alpha} a_{\nu\beta} a_{\rho\gamma} \sigma_{\alpha\beta\gamma}. \qquad (5)$$

It is easy to see that the relativistic equation corresponding to (1) is

$$j_\alpha = \sigma_{\alpha\beta\gamma} F_{\beta\gamma}.$$

On the other hand the energy density quadrivector is written

$$f_\mu = F_{\mu\alpha} j_\alpha. \qquad (7)$$

It results from (6) and (7) that

$$f_\mu = F_{\mu\alpha} \sigma_{\alpha\beta\gamma} F_{\beta\gamma}. \qquad (8)$$

Let us then write the relativistic Euler equations

$$\rho_0 \frac{du_\mu}{d\tau} = -\frac{\partial p}{\partial x_\mu} + F_{\mu\alpha} \sigma_{\alpha\beta\gamma} F_{\beta\gamma}, \qquad (9)$$

where

$$u_\mu = \frac{dx_\mu}{d\tau} \qquad (10)$$

are the components of the velocity quadrivector, and

$$d\tau^2 = c^2\, dt^2 - dx^2 - dy^2 - dz^2. \qquad (11)$$

The equations (9) are the relativistic equations of magneto-hydrodynamics in the case where gravitational forces and those due to viscosity are neglected. But these too may be taken into consideration. We shall assume to that effect that the gravitational field is sufficiently weak for the Fock approximation to be valid [2]. Then the tensor of the energy impulse of the fluid in motion in a gravitational field has the following components :

$$c^2 T^{00} = \rho\left[1 + \frac{1}{c^2}\left(\frac{1}{2}v^2 + p - U\right)\right], \tag{12}$$

$$c^2 T^{0i} = \rho v_i\left[1 + \frac{1}{c^2}\left(\frac{1}{2}v^2 + p - U\right)\right] - \frac{1}{c^2} p_{ik} v_k, \tag{13}$$

$$c^2 T^{ik} = \rho v_i v_k - p_{ik}, \tag{14}$$

where ρ is the density of the matter in motion, v_i are the velocity components and U is the potential of the gravitational field. The equations of motion are then written

$$\nabla_\nu T^{\mu\nu} = F^{\mu\alpha}\sigma^{\alpha\beta\gamma}F^{\beta\gamma}, \tag{15}$$

where

$$\nabla_\nu T^{\mu\nu} = \frac{\partial T^{\mu\nu}}{\partial x^\nu} + \Gamma^\mu_{\alpha\beta} T^{\alpha\beta} + \Gamma^\alpha_{\nu\alpha} T^{\mu\nu}, \tag{16}$$

being the Christoffel symbols of the second kind.

In case of an ideal fluid, $p_{ik} = 0$ $(i, k = 1, 2, 3)$. But if we take the viscosity into account, we may write

$$p_{ik} = -\mu \frac{\partial v_i}{\partial x_k}, \tag{17}$$

$\mu = \rho\nu$ being the dynamic viscosity.

The equations (15) then represent the complete equations of the relativistic magnetohydrodynamics in the Fock approximation.

**** THE END ****

REFERENCES

1. L. MANDELSTAMM et J. TAMM, Math. Ann, 25, p.154, 1926.
2. V. A. FOK, La théorie de l'espace, du temps et de la gravitation, (Theory of Space, Time and Gravitation), in Russian. Moscow, 1955

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